Superconductivity in the Anderson lattice: a finite-*U* slave boson description



2010 J. Phys.: Condens. Matter 22 065702

(http://iopscience.iop.org/0953-8984/22/6/065702)

IOP Publishing JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **22** (2010) 065702 (7pp) doi:10.1088/0953-8984/22/6/065702

# Superconductivity in the Anderson lattice: a finite-$U$ slave boson description

**P D Sacramento**[1], **J Aparício**[1,3] **and G S Nunes**[1,2]

[1] Centro de Física das Interacções Fundamentais, Instituto Superior Técnico, Universidade Técnica de Lisboa, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal
[2] Department of Engineering and Management, ISCTE, Avenida das Forças Armadas, 1600 Lisboa, Portugal

Received 22 September 2009, in final form 21 December 2009
Published 21 January 2010
Online at stacks.iop.org/JPhysCM/22/065702

**Abstract**
Using the Kotliar and Ruckenstein slave boson formalism we consider the finite-$U$ Anderson lattice. We study the appearance of superconductivity as a function of the Coulomb repulsion, density and $f$-level location for s-wave and d-wave pairing symmetries. The results extend previous studies where the infinite-$U$ limit was considered, confirming that superconductivity remains as the Coulomb coupling increases, if the attractive interaction is not weak. Superconductivity disappears for large $U$ as the filling of the heavy particles approaches one, as expected. Since $U$ is finite, superconductivity occurs, in general, for any band-filling being depressed near half-filling, particularly for d-wave symmetry.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Heavy-fermion systems [1] have attracted interest for a long time due to the variety of phases they display, from Fermi-liquid to non-Fermi-liquid, magnetic, superconducting, and even coexistence of magnetism and superconductivity [2]. In the non-ordered phases the quasiparticles often display large effective masses, depending on the regimes, but complications arise probably due to the presence of quantum critical points leading to unusual behavior at small and finite temperatures [3]. A major difficulty associated with these materials is the strong intraband Coulomb repulsion, $U$, in the narrow-band associated with the heavy masses. In general, this scale is the largest one and various methods have been used to take care of the restricted motion of these particles, such as the slave boson method, originally introduced to deal with a limit where the Coulomb coupling is infinite [4, 5]. A generalization of the method to finite $U$ was first introduced at the level of the Hubbard model [6] (where the same kind of local repulsion occurs) and later generalized to the Anderson model [7]. The method has been generalized to include spin-rotation invariance [8, 9]. The method has also been applied in the context of high-temperature superconductors [10, 11]. The Anderson model is generally accepted as appropriate to describe heavy-fermion systems [12]. Using slave boson descriptions both magnetic and superconducting instabilities have been successfully studied [7, 13–17], as have their competition/coexistence [18, 19].

In this work we will focus on the superconducting phase of these materials. Using Coleman's [5] slave boson formalism together with a large-$N$ approach, it was proposed [20] that slave boson fluctuations can provide an effective attraction between the electrons, to leading order in $1/N$. Later, a calculation of the quasiparticle–quasiparticle scattering amplitude to order $1/N^2$ revealed an effective attractive interaction in the p and d channels, which was interpreted as a manifestation of the RKKY interaction, showing that spin fluctuations are an important mechanism [21]. A magnetic origin was also proposed in [22–24] and argued for in [2, 25]. The magnetic and superconducting instabilities of the periodic Anderson model were also studied using a RPA method [26]. An effective antiferromagnetic interaction arises between the local moments which can then be decoupled in various ways, including terms that lead to pairing [27]. The magnetic interaction is in general complex and involves the hybridization between the local $f$-electrons and the $c$-electrons being of the order of $V^4$, where $V$ is the hybridization between the $c$-electrons (nearly free, extended) and the $f$-electrons (localized and strongly correlated). The complexity of the underlying mechanism was successfully overcome using as input both band structure calculations and neutron scattering data [28, 29].

Whatever the origin of the effective interaction, various studies have been carried out imposing a phenomenological attractive interaction between nearest-neighbor heavy electrons.

[3] Present address: Instituto de Física Teórica CSIC/UAM C-XVI, Universidad Autónoma de Madrid, E-28049 Madrid, Spain.

 

Experimental results for the specific heat in the superconducting phase show that heavy fermions are involved in the pairing. Due to the hybridization, the light (conduction) and heavy (narrow-band) electrons are mixed into bands of heavy quasiparticles with a predominantly $f$-electron character and, as shown before [30], it is the pairing between the $f$-electrons that is responsible for superconductivity. Note however that the bare $f$-electrons are dispersionless and it is the result of the hybridization that heavy quasiparticles become the pairing quasiparticles. Therefore, in this work we will add to the Anderson lattice Hamiltonian a phenomenological attractive interaction between the nearest-neighbor $f$-electron densities, with magnitude $J < 0$, since these particles have the largest weight in the heavy quasiparticles and since we will study both the transition temperature and the superconducting phase ($T = 0$). Clearly via the hybridization the $c$-electrons will also pair. We will come back to this point later.

This problem was studied before in the case of finite $U$ using a perturbative approach [31] and in the $U = \infty$ limit a slave boson method [13] and the X-boson method [14] were used. It was predicted in the perturbative approach that for large enough $U$ superconductivity disappears. Also, in a regime where the heavy electron density is larger than the light electron density, superconductivity disappears fast as the band-filling increases beyond half-filling, due to the saturation of the heavy electron band. However, using the slave boson method it was found [13] that, for infinite coupling, superconductivity prevails in a regime where the heavy electron filling is smaller than the light electron filling. This was confirmed by the X-boson approach [14]. In these works the relative stabilities of the various pairing symmetries were studied, and the basic conclusion is that the critical temperatures for the various symmetries are in general similar and the dominant one varies as the parameters of the model change (see however [18, 19] where a quantum phase transition is predicted for d-wave symmetry when antiferromagnetism and superconductivity coexist). In this work we consider the case of a finite Coulomb coupling using the Kotliar and Ruckenstein slave boson approach [6]. We confirm the stability of the superconducting order as $U$ grows, in agreement with the infinite $U$ results previously obtained.

## 2. The model and slave boson description

The model we study is the Anderson lattice, with $N_s$ sites, where two sets of electrons, $c$-electrons described by the band $\epsilon_k$, and $f$-electrons described by the energy $\epsilon_f$, are hybridized with an amplitude $V$. The Hamiltonian is given by [7]

$$\begin{aligned}H &= \sum_{k,\sigma}\epsilon_k c^\dagger_{k,\sigma}c_{k,\sigma} + \sum_{k,\sigma}\epsilon_f f^\dagger_{k,\sigma}f_{k,\sigma} + U\sum_i d^\dagger_i d_i \\ &+ V\sum_{i,\sigma}(c^\dagger_{i,\sigma}f_{i,\sigma}Z_{i,\sigma} + Z^\dagger_{i,\sigma}f^\dagger_{i,\sigma}c_{i,\sigma}) \\ &+ \sum_i \lambda_i(e^\dagger_i e_i + d^\dagger_i d_i + p^\dagger_{i,\uparrow}p_{i,\uparrow} + p^\dagger_{i,\downarrow}p_{i,\downarrow} - 1) \\ &+ \sum_{i,\sigma}\lambda_{i,\sigma}(f^\dagger_{i,\sigma}f_{i,\sigma} - p^\dagger_{i,\sigma}p_{i,\sigma} - d^\dagger_i d_i). \qquad (1)\end{aligned}$$

In general, the heavy-fermion systems are 3d-systems even though in some cases quite anisotropic. For simplicity and to reduce computational time we will consider here a two-dimensional system since the results are qualitatively the same, as shown before [13, 18, 19]. There is a Coulomb repulsion $U$ if two $f$-electrons are located at the same site. We associate four bosons to the various states $f$-electrons can occupy [6]. The bosons $e, d$ are associated with empty and doubly occupied sites, respectively, and the bosons $p_\sigma$ with a singly occupied site with spin component $\sigma$. There is an enlargement of the Hilbert space and restrictions must be implemented at each site, $e^\dagger_i e_i + d^\dagger_i d_i + p^\dagger_{i,\uparrow}p_{i,\uparrow} + p^\dagger_{i,\downarrow}p_{i,\downarrow} = 1$, and $f^\dagger_{i,\sigma}f_{i,\sigma} = p^\dagger_{i,\sigma}p_{i,\sigma} + d^\dagger_i d_i$, through Lagrange multipliers $\lambda_i$ and $\lambda_{i,\sigma}$, respectively. In the physical subspace the operators $f_{i,\sigma}$ are replaced by $f_{i,\sigma}Z_{i,\sigma}$ where $Z_{i,\sigma} = (1 - d^\dagger_i d_{i,\sigma} - p^\dagger_{i,\sigma}p_{i,\sigma})^{-1/2}(e^\dagger_i p_{i,\sigma} + p_{i,-\sigma}d_i)(1 - e^\dagger_i e_{i,\sigma} - p^\dagger_{i,-\sigma}p_{i,-\sigma})^{-1/2}$. The usual procedure consists in taking a mean-field approach where we assume the slave bosons to be condensed. In the cases of paramagnetic or ferromagnetic solutions we take $Z^\dagger_{i,\sigma} = Z_{i,\sigma} = Z_\sigma$, $e^\dagger_i = e_i = e$, $d^\dagger_i = d_i = d$, $p^\dagger_{i,\sigma} = p_{i,\sigma} = p_\sigma$. The paramagnetic solution is described by $p_\uparrow = p_\downarrow = p$. As a consequence $\lambda_\uparrow = \lambda_\downarrow = \bar\lambda$ and $Z_\uparrow = Z_\downarrow = Z$.

We study the appearance of superconductivity neglecting any form of magnetic order. We take a usual mean-field approximation where the Anderson Hamiltonian with the pairing interaction term added can be simplified to

$$\begin{aligned}H_{\mathrm{MF}} &= \sum_{k,\sigma}(\epsilon_k - \mu)c^\dagger_{k,\sigma}c_{k,\sigma} + \sum_{k,\sigma}(\tilde\epsilon_f - \mu)f^\dagger_{k,\sigma}f_{k,\sigma} \\ &+ V\sum_{k,\sigma}Z(c^\dagger_{k,\sigma}f_{k,\sigma} + f^\dagger_{k,\sigma}c_{k,\sigma}) \\ &+ Z^2\sum_k(\Delta\eta_k f^\dagger_{k,\uparrow}f^\dagger_{-k,\downarrow} + \Delta^*\eta_k f_{-k,\downarrow}f_{k,\uparrow}) \\ &+ UN_s d^2 + N_s\lambda(e^2 + d^2 + 2p^2 - 1) \\ &- N_s\sum_\sigma\bar\lambda(p^2 + d^2) - N_s\frac{|\Delta|^2}{J} \qquad (2)\end{aligned}$$

where $\epsilon_k = -2t(\cos k_x + \cos k_y)$ in two dimensions, $\tilde\epsilon_f = \epsilon_f + \bar\lambda$ and we have added the chemical potential $\mu$ to fix the total electronic density $n = n_c + n_f$ with $n_c = 1/N_s\sum_{k,\sigma}c^\dagger_{k\sigma}c_{k\sigma}$, $n_f = 1/N_s\sum_{k,\sigma}f^\dagger_{k\sigma}f_{k\sigma}$. The pairing symmetry is selected as either an extended s-wave or a d-wave, which for a two-dimensional square lattice are obtained using $\eta_k = 2\cos k_x + 2\cos k_y$ or $\eta_k = 2\cos k_x - 2\cos k_y$, respectively. In some heavy-fermion systems the symmetry appears to be d-wave but often it has not been unambiguously determined. Therefore we consider here both d-wave and extended s-wave since the strong local repulsion prevents local s-wave pairing between the $f$-electrons. The generalization of the bands and pairings to the 3d case is straightforward [32, 13]. The mean-field treatment of the attractive interaction involves the usual decoupling of destruction and creation operators but we associate a boson operator, $Z$, with every $f$ operator in order to prevent double occupancy at the $f$ sites (a similar procedure has been used before in the context of the $t$–$J$ model [33] and in the Anderson lattice [13]). As discussed before [13] the presence of the slave bosons in the pairing term is important to describe the experimental results since in the infinite $U$ limit it guarantees that the superconducting order vanishes as $n_f \to 1$. The same occurs in the finite $U$ formalism. For large $U$ one expects that $Z^2 \to (1-n_f)/(1-n_f/2)$, which vanishes as $n_f \to 1$, as intended. In this limit we also obtain that $p^2 = n_f/2$ and $e^2 = 1-n_f$ and the effective coupling decreases, as expected.

The procedure now consists of determining the mean-field values of the condensed bosons and the superconducting order parameter using the Hellmann–Feynmann theorem by taking derivatives of the Hamiltonian with respect to these parameters. These derivatives lead to mean-field equations that have to be solved self-consistently. Defining

$$F_{V,b} = V\frac{\partial Z}{\partial b}\sum_{k,\sigma}(\langle c^\dagger_{k,\sigma} f_{k,\sigma}\rangle + \langle f^\dagger_{k,\sigma} c_{k,\sigma}\rangle)$$

and

$$F_{\Delta,b} = \frac{\partial Z^2}{\partial b}\sum_k(\Delta\eta_k\langle f^\dagger_{k,\uparrow} f^\dagger_{-k,\downarrow}\rangle + \Delta^*\eta_k\langle f_{-k,\downarrow} f_{k,\uparrow}\rangle)$$

where $b = p, d, e$, this leads to

$$F_{V,p} + F_{\Delta,p} + 2N_s p(\lambda - \bar{\lambda}) = 0 \quad (3)$$

$$F_{V,d} + F_{\Delta,d} + 2N_s d(U + \lambda - 2\bar{\lambda}) = 0 \quad (4)$$

$$F_{V,e} + F_{\Delta,e} + 2N_s e\lambda = 0 \quad (5)$$

$$Z^2\sum_k \eta_k\langle f_{-k,\downarrow} f_{k,\uparrow}\rangle - \frac{N_s\Delta}{J} = 0 \quad (6)$$

together with the restrictions $N_s(e^2 + d^2 + p_\uparrow^2 + p_\downarrow^2 - 1) = 0$ and $\sum_k\langle f^\dagger_{k,\sigma} f_{k,\sigma}\rangle - N_s(p_\sigma^2 + d^2) = 0$.

## 3. Bogoliubov–de Gennes equations

The solution of these equations requires the evaluation of the various fermion operator averages. This can be done in various ways such as using Green function methods, directly diagonalizing the Hamiltonian operator by performing a rotation of the fermionic operators as indicated in [19] or through the use of Bogoliubov–de Gennes (BdG) equations. These can be obtained in the standard way by defining

$$c_{k,\sigma} = \sum_n[u_n^c(k,\sigma)\gamma_n - \sigma v_n^c(k,\sigma)\gamma_n^\dagger] \quad (7)$$

and

$$f_{k,\sigma} = \sum_n[u_n^f(k,\sigma)\gamma_n - \sigma v_n^f(k,\sigma)\gamma_n^\dagger] \quad (8)$$

and the vector

$$\psi_n^T = (u_n^c(k,\uparrow),\quad u_n^f(k,\uparrow),\quad v_n^c(k,\downarrow),\quad v_n^f(k,\downarrow)). \quad (9)$$

The BdG equations can then be written as $\mathcal{H}\psi_n = \epsilon_n\psi_n$ where the matrix $\mathcal{H}$ is given by

$$\mathcal{H} = \begin{pmatrix} \epsilon_k - \mu & VZ & 0 & 0 \\ VZ & \tilde{\epsilon}_f - \mu & 0 & Z^2\Delta\eta_k \\ 0 & 0 & -(\epsilon_k - \mu) & -VZ \\ 0 & Z^2\Delta\eta_k & -VZ & -(\tilde{\epsilon}_f - \mu) \end{pmatrix} \quad (10)$$

The solution of these equations yields the energy eigenvalues and the functions $u, v$ needed to calculate any operator thermodynamic average.

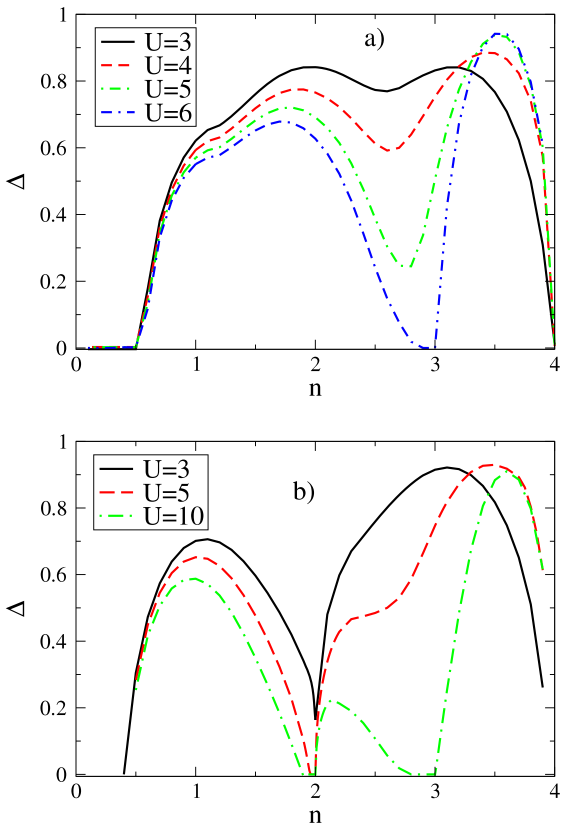


**Figure 1.** $\Delta$ as a function of band-filling $n$ for various Coulomb couplings for (a) s-wave and (b) d-wave symmetries ($T = 0$).

## 4. Zero-temperature results

### 4.1. Order parameter

We consider first the self-consistent appearance of superconducting order in the system by considering the zero-temperature order parameter. In figure 1 we show the order parameter $\Delta$ as a function of band-filling for various values of the Coulomb repulsion, $U$, for a typical set of values $V = 1, \epsilon_f = -0.5$ and for an attractive interaction between the $f$-electrons, $J = -3$. We take as the energy unit $t = 1$. We consider both extended s-wave and d-wave symmetries. For these parameters, when turning off $J$, the system is in a metallic phase. Turning on $J$ the system may become superconducting. For intermediate values of $U$ (such as $U = 3$) superconductivity is present in a wide range of band-fillings. At low-fillings the order parameter vanishes because there are not enough electrons to pair. As $U$ increases there is a depression in the vicinity of half-filling ($n = 2$) which is particularly noticeable in the d-wave case. In the s-wave case and for $U = 6$ there is a total suppression of $\Delta$ near $n = 3$ but then it becomes finite as $n$ grows further.

In order to better understand these results we present in figure 2 the values of the Bose condensates for the s-wave case (the results for d-wave symmetry are very similar). As the band-filling increases the number of empty sites decreases to zero and the number of doubly occupied sites increases up to saturation if $U$ is small ($U = 3$). However, if $U$ is

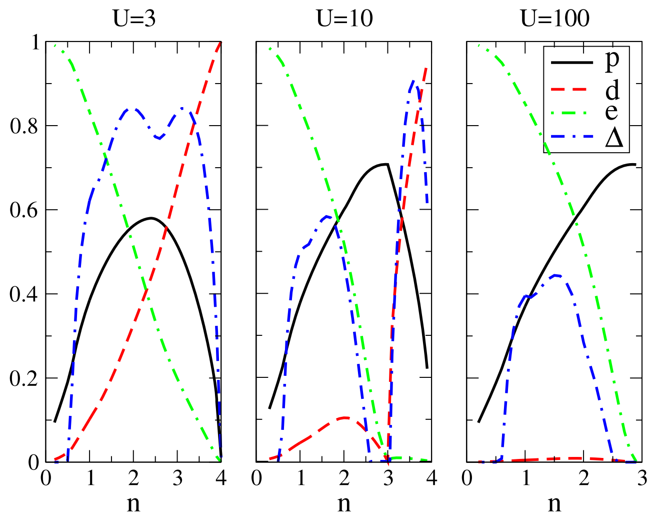


**Figure 2.** Various parameters $p, d, e, \Delta$ as a function of band-filling for $U = 3$, 10, 100 for s-wave symmetry ($T = 0$).

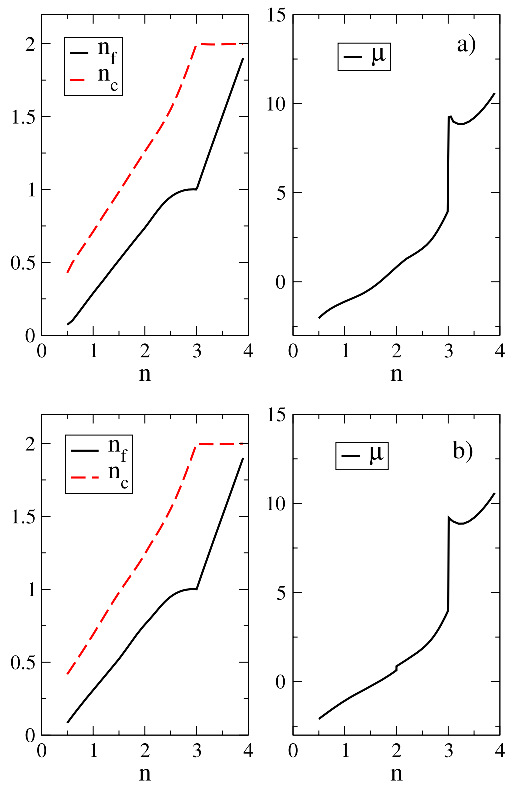


**Figure 3.** $n_f$, $n_c$, and $\mu$ for (a) s-wave and (b) d-wave symmetries and $U = 10$.

large the number of doubly occupied sites is very small, as expected. For large $U$ at large band-filling all the sites are singly occupied since also the number of empty sites tends to zero. However, for $n > 3$ the number of doubly occupied sites increases implying an occupation of the upper Hubbard band. The figure also shows the superconducting order parameter, $\Delta$. As $U$ increases $\Delta$ vanishes for intermediate values of the band-filling, as expected. Note that even for $U = 100$ superconductivity prevails up to $n \sim 2.5$.

One would expect from previous treatments that, for large $U$, superconductivity should disappear near half-filling ($n = 2$). The reasoning is that at half-filling one expects that $n_f = 1$ and the motion of the Cooper pairs is inhibited. Note however the strong increase of $\Delta$ for $U = 10$ as the band-filling $n > 3$ (the same happens for $U = 100$, not shown). The values of $n_f, n_c, \mu$ are shown in detail for the case of $U = 10$ in figure 3 for both symmetries. We see that when $n = 3$ the filling of the $c$-electrons saturates. Also, we see that for $n \sim 2.5$ the filling of the $f$-electrons is already very close to $n_f = 1$. Therefore we expect that $\Delta \sim 0$ at this point. We see also that in the d-wave case there is a jump in the chemical potential at $n = 2$ which indicates an insulator regime and a consequent vanishing of $\Delta$. Also, we see clearly that there is an abrupt change of $\mu$ at $n = 3$. At this point there is a jump to the upper Hubbard band with a high energy cost. The reappearance of superconductivity is due to pairing between electrons in the upper Hubbard band. It is therefore a high energy phase and the non-superconducting phase has a lower free energy. We have compared the free energies of the three phases and have found that in this high filling regime the $\Delta = 0$ phase has the lowest energy. In the other regimes either the s-wave or the d-wave phase are the most stable (we note that the non-superconducting phase is usually the second lowest energy phase).

The stability of superconductivity at increasing Coulomb couplings is in agreement with the results obtained by the infinite-$U$ slave boson method [13] and the X-boson method [14]. The results are however in apparent disagreement with a perturbative calculation [31], as previously emphasized in the infinite-$U$ limit. We note however that the results are obtained in different parameter regimes. In [31] the regime is such that $n_f > n_c$. As the Coulomb coupling $U$ increases $n_f \to 1$ preventing the motion of the Cooper pairs and effectively suppressing superconductivity. In this work, we are in a regime where $n_f < n_c$ and therefore while $n_f < 1$ there is room for a non-vanishing order parameter.

### *4.2. Spectrum*

Complementary information about the system behavior may be obtained by the excitations spectrum. Due to the superconducting order the spectrum has a gap that tracks the behavior of $\Delta$ in the s-wave case (in the d-wave case this occurs away from the nodal directions). The gap (defined by the lowest positive energy eigenvalue) has a maximum at half-filling which decreases as $U$ increases. The gap is the result of various effects like the superconducting order and the hybridization. There are two other peaks, one slightly below quarter-filling and the other for small values of $U$ (say $U = 3$) at large band-fillings (say $n \sim 3.5$). These peaks in the gap agree with the structure seen in the order parameter.

Changing the value of the bare $f$-level does not change qualitatively the results. The gap has a similar structure. There is a peak with a value independent of $\epsilon_f$ still locked at half-filling, but the other two peaks are slightly displaced. Also their values change. The order parameter changes slightly, accordingly shifting to slightly smaller band-filling values as the location of the $f$-level lowers in energy.

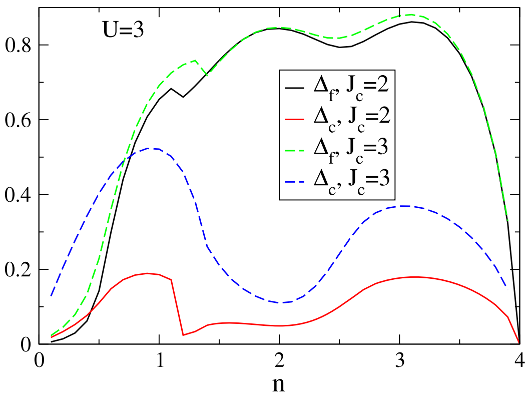


**Figure 4.** $\Delta_f$ and $\Delta_c$ as a function of $n$ for s-wave symmetry for two different values of $J_c$ and $U = 3$.

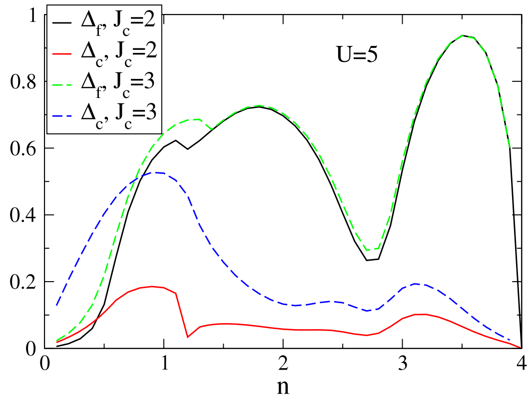


**Figure 5.** $\Delta_f$ and $\Delta_c$ as a function of $n$ for s-wave symmetry for two different values of $J_c$ and $U = 5$.

## 5. Pairing between the *c*-electrons

Even though the bare $c$-electrons are free and therefore have a high mobility and low effective masses, through the coupling to the heavy $f$-electrons, they may also pair. The results of the previous section were obtained with no explicit pairing of the $c$-electrons. Here we consider the possibility of $c$-electron pairing in the mean-field Hamiltonian with the inclusion of a second coupling constant $J_c$. Phenomenologically we may consider this pairing as adding a term in the Hamiltonian leading to

$$\begin{aligned} H_{\mathrm{MF}} = & \sum_{k,\sigma}(\epsilon_k - \mu)c^\dagger_{k,\sigma}c_{k,\sigma} + \sum_{k,\sigma}(\tilde{\epsilon}_f - \mu)f^\dagger_{k,\sigma}f_{k,\sigma} \\ & + V\sum_{k,\sigma} Z(c^\dagger_{k,\sigma}f_{k,\sigma} + f^\dagger_{k,\sigma}c_{k,\sigma}) \\ & + Z^2\sum_k(\Delta\eta_k f^\dagger_{k,\uparrow}f^\dagger_{-k,\downarrow} + \Delta^*\eta_k f_{-k,\downarrow}f_{k,\uparrow}) \\ & + UN_s d^2 + N_s\lambda(e^2 + d^2 + 2p^2 - 1) \\ & - N_s\sum_\sigma \bar{\lambda}(p^2 + d^2) - N_s\frac{|\Delta|^2}{J} \\ & + \sum_k(\Delta_c\eta^c_k c^\dagger_{k,\uparrow}c^\dagger_{-k,\downarrow} + \Delta^*_c\eta^c_k c_{-k,\downarrow}c_{k,\uparrow}) \\ & - N_s\frac{|\Delta_c|^2}{J_c}. \end{aligned} \tag{11}$$

This extra term leads to modified BdG equations where the Hamiltonian operator is now given by

$$\mathcal{H} = \begin{pmatrix} \epsilon_k - \mu & VZ & \Delta_c\eta^c_k & 0 \\ VZ & \tilde{\epsilon}_f - \mu & 0 & Z^2\Delta\eta_k \\ \Delta_c\eta^c_k & 0 & -(\epsilon_k - \mu) & -VZ \\ 0 & Z^2\Delta\eta_k & -VZ & -(\tilde{\epsilon}_f - \mu) \end{pmatrix}. \tag{12}$$

Since there is no local repulsive coupling between the $c$-electrons, we consider only a local s-wave pairing of the $c$-electrons: $\eta^c_k = 1$.

In figures 4–7 we show the results for the two pairing amplitudes $\Delta$, $\Delta_c$. In the case of s-wave symmetry it can be seen that qualitatively there is not much difference in the curves for $\Delta_f$, with respect to the results of the previous section and therefore changing $J_c$ does not strongly affect the result. The

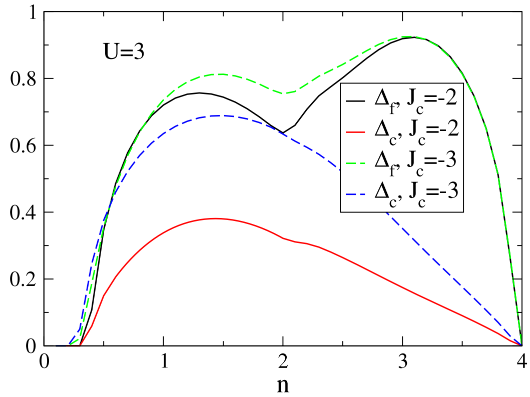


**Figure 6.** $\Delta_f$ and $\Delta_c$ as a function of $n$ for d-wave symmetry for two different values of $J_c$ and $U = 3$.

$c$-pairing order parameter is smaller but extends to lower band-fillings, as expected. For d-wave symmetry, the dip in the $\Delta_f$ curves at around $n = 2$ is less pronounced. Note that in this case $\Delta_c$ also vanishes at small band-fillings. The two order parameters follow similar trends in the case of s-wave symmetry, displaying two maxima as a function of the band-filling. In the case of d-wave pairing the peak at larger band-fillings is absent in $\Delta_c$. We note however that the curves for $p$, $d$ and $e$ remain qualitatively similar with the addition of the extra pairing term. The boson mean-field condensed values do not depend strongly on the symmetry of the order parameter.

In what follows we have therefore neglected such an additional term to the Hamiltonian. That is, we have considered $J_c = 0$, since it does not affect $\Delta$ in an important way.

## 6. Critical temperature

It is perhaps more instructive to analyze the dependence of the critical temperature as a function of the various parameters. In BCS theory there is a close relation between the superconducting order parameter and the critical temperature, specifically, for a single-band model they are just proportional. We recall however that in the Anderson model there are two bands and superconductivity is competing with the strong correlations and the hybridization between the bands.

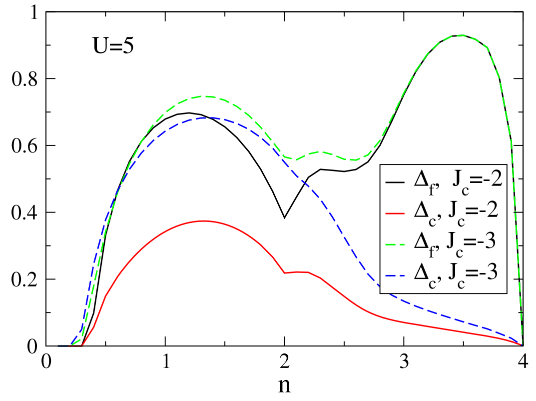


**Figure 7.** $\Delta_f$ and $\Delta_c$ as a function of $n$ for d-wave symmetry for two different values of $J_c$ and $U = 5$.

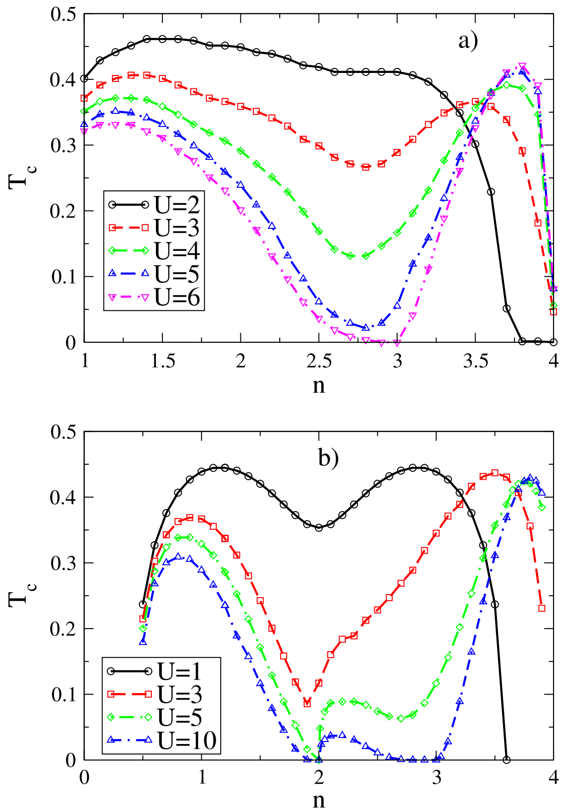


**Figure 8.** Superconducting critical temperature as a function of $n$ for (a) s-wave and (b) d-wave symmetries.

In figure 8 we show the critical temperature as a function of band-filling for various Coulomb couplings for both symmetries. The trend is similar to the results obtained for the order parameter. There is a significant depression in the vicinity of half-filling (actually $n_f$ close to 1) with a restoration of superconductivity for larger band-fillings, being finally depressed when the bands are full. We also show in figure 9 the dependence of $T_c$ for s-wave symmetry as a function of $U$ where the flattening of the critical temperature is noticeable emphasizing its stability as $U$ increases. Also, we consider the dependence with the hybridization. We see that for small values of $U$ and at small $V$ the critical temperature decreases, goes through a maximum and finally decreases for

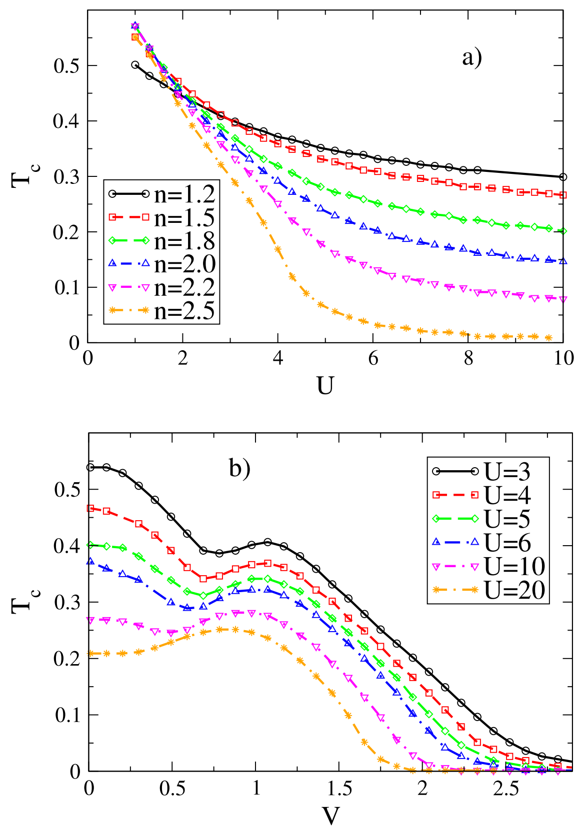


**Figure 9.** Superconducting critical temperature as a function of (a) $U$ and (b) $V$ for s-wave symmetry.

large enough hybridization. We note that for larger values of $U$ there is a tendency for the critical temperature to increase with $V$ for small values, in agreement with the results obtained [13] with the slave boson method for $U = \infty$. This shows some agreement between Coleman's method and the finite-$U$ method. We note however that, as mentioned above, the finite-$U$ method, at least for the parameters chosen here, yields values of $n_f$ that grow more slowly with the band-filling, with the consequence that by increasing the value of $U$ superconductivity prevails for larger band-fillings.

One expects that a weak attractive interaction may not be enough to yield superconductivity. This is shown for s-wave symmetry in figure 10 where the critical temperature as a function of $n$ is considered for $U = 3, 10$ and smaller values of the attractive coupling $J = -2, -1.5, -1, -0.5$. As expected, as $|J|/U$ decreases the superconducting region decreases even for $U = 3$. Note that at $n = 2$ and smaller values of $J$ superconductivity vanishes. At these points the gap has a contribution from a hybridization gap as discussed before in [14].

## 7. Summary

In this work we have shown that in the finite-$U$ slave boson description [6] of the Anderson lattice, superconductivity prevails in general for all values of the Coulomb repulsion. In the case of extended s-wave symmetry either the superconducting order parameter or the critical temperature become very small, close to a band-filling of the order of

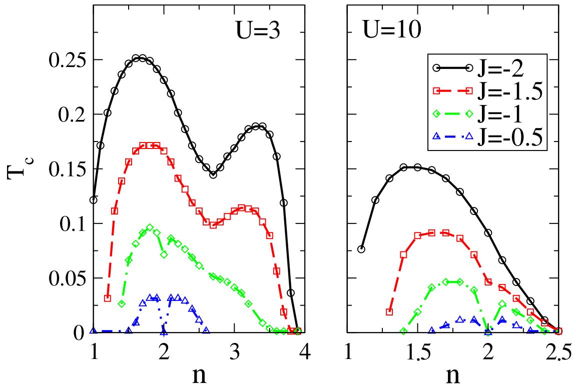


**Figure 10.** Superconducting critical temperature as a function of $n$ for s-wave symmetry for different values of $J$.

$n = 3$, where $n_f \to 1$ and the chemical potential has a discontinuity characteristic of insulating behavior. Beyond this regime there is a high energy regime where pairing occurs in the upper Hubbard band. This regime is not energetically favorable at zero temperature. In the case of d-wave symmetry something similar occurs, but in addition, there is another vanishing point at half-filling ($n = 2$) where there is also a jump in the chemical potential.

Heavy-fermion systems also display magnetic phases, as stated above. Here we have only considered the superconducting phase. Previous studies considered the possibility of magnetic phases (without considering the competition with superconductivity) and have shown that close to half-filling and quarter-filling an antiferromagnetic phase prevails [7, 15, 34] while in other cases one expects spiral phases or ferromagnetic phases [17]. The competition/coexistence of antiferromagnetism was also considered previously in the case of $U = \infty$ and it was found that close to half-filling antiferromagnetism is the most stable phase while at lower band-fillings superconductivity prevails. Also, the effect of applying pressure leads to the disappearance of antiferromagnetism [19]. The results obtained here hold in a regime where superconductivity is observed.

## Acknowledgment

We acknowledge support from the ESF Science Program INSTANS 2005–2010 and support from FCT (Portugal) through the grant PTDC/FIS/64926/2006.

## References

[1] Hewson A C 1997 *The Kondo Problem to Heavy-Fermions* (Cambridge: Cambridge University Press)
[2] Mathur N D *et al* 1998 *Science* **394** 39
[3] Steglich F *et al* 1996 *J. Phys.: Condens. Matter* **8** 9909
von Lohneysen H *et al* 1996 *J. Phys.: Condens. Matter* **8** 9689
Schroeder A *et al* 2000 *Nature* **407** 351
[4] Barnes S E 1976 *J. Phys. F: Met. Phys.* **6** 1375
Barnes S E 1977 *J. Phys. F: Met. Phys.* **7** 2637
[5] Coleman P 1984 *Phys. Rev.* B **29** 3035
[6] Kotliar G and Ruckenstein A 1986 *Phys. Rev. Lett.* **57** 1362
[7] Dorin V and Schlottmann P 1992 *Phys. Rev.* B **46** 10800
[8] Li T C, Wölfle P and Hirschfeld P J 1989 *Phys. Rev.* B **40** 6817
Frésard R and Wölfle P 1992 *Int. J. Mod. Phys.* B **6** 685
[9] Frésard R and Wölfle P 1992 *J. Phys.: Condens. Matter* **4** 3625
[10] Lee P A, Nagaosa N and Wen X G 2006 *Rev. Mod. Phys.* **78** 17
[11] Raczskowski M, Frésard R and Oles A M 2006 *Phys. Rev.* B **73** 174525
[12] Millis A J and Lee P A 1987 *Phys. Rev.* B **35** 3394
[13] Araújo M A N, Peres N M R, Sacramento P D and Vieira V R 2000 *Phys. Rev.* B **62** 9800
[14] Nunes L H C M, Figueira M S and de Mello E V L 2003 *Phys. Rev.* B **68** 134511
[15] Doradzinski R and Spalek J 1998 *Phys. Rev.* B **58** 3293
[16] Aparício J, Nunes G S and Sacramento P D 2009 *J. Phys. Conf. Ser.* **150** 042144
[17] Möller B and Wölfle P 1993 *Phys. Rev.* B **48** 10320
[18] Araújo M A N, Peres N M R and Sacramento P D 2001 *Phys. Rev.* B **65** 012503
[19] Sacramento P D 2003 *J. Phys.: Condens. Matter* **15** 6285
[20] Lavagna M, Millis A J and Lee P A 1987 *Phys. Rev. Lett.* **58** 266
[21] Houghton A, Read N and Won H 1988 *Phys. Rev.* B **37** 3782R
[22] Grewe N, Pruschke T and Keiter H 1988 *Z. Phys.* B **71** 75
[23] Coleman P and Andrei N 1989 *J. Phys.: Condens. Matter* **1** 4057
[24] Welslau B and Grewe N 1992 *Ann. Phys.* **1** 214
[25] Sato N K *et al* 2001 *Nature* **410** 340
[26] Peres N M R and Araújo M A N 2002 *J. Phys.: Condens. Matter* **14** 5575
[27] Ubbens M U and Lee P A 1992 *Phys. Rev.* B **46** 8434
[28] Norman M R 1987 *Phys. Rev. Lett.* **59** 232
[29] Norman M R 1988 *Phys. Rev.* B **37** 4987
[30] Tachiki M and Maekawa S 1984 *Phys. Rev.* B **29** 2497
[31] Romano A, Noce C and Micnas R 1997 *Phys. Rev.* B **55** 12640
[32] Bastide C and Lacroix C 1988 *J. Phys. C: Solid State Phys.* **21** 3557
[33] Ruckenstein A E, Hirschfeld P J and Appel J 1987 *Phys. Rev.* B **36** 857
[34] Dorin V and Schlottmann P 1993 *J. Appl. Phys.* **73** 5400